$\square$ Max. : 100 Marks
Time : 01:00-04:00

## SECTION -A

## Answer all questions

( $10 \times 2=20$ Marks $)$

1. Define the marginal distribution of $X$ given the joint probability mass function.
2. Define $\mathrm{E}[\mathrm{X} / \mathrm{Y}=\mathrm{y}]$.
3. Define discrete uniform distribution.
4. 10 coins are thrown simultaneously. Find the probability of getting atleast seven heads.
5. Under what conditions Poisson distribution is a limiting case of Binomial distribution.
6. In a book of 520 pages, 390 typo-graphical errors occur. Assuming Poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.
7. Define Geometric distribution.
8. Under what conditions Negative Bionomial distribution is a limiting case of Poisson distribution.
9. Obtain mean of Hyper Geometric distribution.
10. Define Multinomial distribution.

## SECTION-B

Answer any FIVE questions.
( $5 \times 8=40$ marks $)$
11. The joint probability distribution of two random variables X and Y is given by
$\mathrm{P}(\mathrm{X}=0, \mathrm{Y}=1)=1 / 3, \mathrm{P}(\mathrm{X}=1, \mathrm{Y}=-1)=1 / 3$ and $\mathrm{p}(\mathrm{X}=1, \mathrm{Y}=1)=1 / 3$
find (i) marginal distributions of X and Y
(ii) $\mathrm{p}(\mathrm{X}=1 / \mathrm{Y}=1)$
(iii) $\mathrm{E}[\mathrm{X}]$
(iv) $\mathrm{E}[\mathrm{X} / \mathrm{Y}=1]$
12. Obtain recurrence relationship for Binomial distribution.
13. A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 15. Calculate the proportion of days on which (i) neither car is used and (ii) the proportion of days on which some demand is refused.
14. Explain memoryless property of Geometric distribution.
15. Obtain MGF of trinomial distribution.
16. Let $X_{1}, X_{2}$ be independent random variables each having geometric distribution, $q^{k} p$, $\mathrm{k}=0,1,2, \ldots \ldots$ show that the conditional distribution of $\mathrm{X}_{1}$ given $\mathrm{X}_{1}+\mathrm{X}_{2}$ is uniform.
17. In a Poisson frequency distribution, frequency corresponding to 3 successes is $2 / 3$ times frequency corresponding to 4 successes. Find mean and standard deviation of the distribution.
18. Let X and Y be two random variables each taking three values $-1,0$ and 1 and having the joint probability distributions

| $\mathrm{X} \backslash \mathrm{Y}$ | -1 | 0 | 1 |
| ---: | :---: | :--- | :--- |
| -1 | $1 / 27$ | $1 / 9$ | $2 / 27$ |
| 0 | $1 / 9$ | $5 / 27$ | $1 / 3$ |
| 1 | 0 | $1 / 9$ | $1 / 27$ |

Find $V(Y / X=-1)$.

## SECTION-C

## Answer any Two questions.

( $2 \times 20=40$ marks )
19. The joint probability distribution of X and Y is given by the following table.

| $\mathrm{X} \mid \mathrm{Y}$ | 1 | 3 | 9 |
| :---: | :---: | :---: | ---: |
| 2 | $1 / 8$ | $1 / 24$ | $1 / 12$ |
| 4 | $1 / 4$ | $1 / 4$ | 0 |
| 6 | $1 / 8$ | $1 / 24$ | $1 / 12$ |

(i) Find marginal distribution of Y
(ii) Find conditional distribution of $\mathrm{Y} / \mathrm{X}=2$
(iii) Find $\operatorname{COV}(\mathrm{X}, \mathrm{Y})$
(iv) Are X and Y independent?
20. Derive the recurrence relation for the moments of Poisson distribution. Hence obtain the first four central moments.
21. (a) Obtain the density function of a Poisson distribution as a limiting case of Binomial distribution.
(b) Obtain MGF of Negative Binomial distribution and hence obtain its mean and variance.
22. (a) Explain Hyper- Geometric distribution.
(b) If $X_{1}$ and $X_{2}$ are independent Poisson variates with parameters $\lambda_{1}$ and $\lambda_{2}$ respectively, find the distribution of $X_{1}=r$ given $X_{1}+X_{2}=n$.

